Program : **B.Tech**

Subject Name: **Structural Analysis**

Subject Code: **CE-403**

Semester: **4th**

Unit-3
Virtual work and Energy Principles:

Indeterminate Structures-II: Analysis of beams and frames by slope Deflection method, Column Analogy method.

Slope deflection Method

Introduction.

As pointed out earlier, there are two distinct methods of analysis for statically indeterminate structures depending on how equations of equilibrium, load displacement and compatibility conditions are satisfied: 1) force method of analysis and (2) displacement method of analysis. In the last module, force method of analysis was discussed. In this module, the displacement method of analysis will be discussed. In the force method of analysis, primary unknowns are forces and compatibility of displacements is written in terms of pre-selected redundant reactions and flexibility coefficients using force displacement relations. Solving these equations, the unknown redundant reactions are evaluated. The remaining reactions are obtained from equations of equilibrium.

As the name itself suggests, in the displacement method of analysis, the primary unknowns are displacements. Once the structural model is defined for the problem, the unknowns are automatically chosen unlike the force method. Hence this method is more suitable for computer implementation. In the displacement method of analysis, first equilibrium equations are satisfied. The equilibrium of forces is written by expressing the unknown joint displacements in terms of load by using load displacement relations. These equilibrium equations are solved for unknown joint displacements. In the next step, the unknown reactions are computed from compatibility equations using force displacement relations. In displacement method, three methods which are closely related to each other will be discussed.

1. Slope-Deflection Method

2. Moment Distribution Method

3. Direct Stiffness Method

In this module first two methods are discussed and direct stiffness method is treated in the next module. All displacement methods follow the above general procedure. The Slope-deflection and moment distribution methods were extensively used for many years before the compute era. After the revolution occurred in the field of computing only direct stiffness method is preferred.

Degrees of freedom

In the displacement method of analysis, primary unknowns are joint displacements which are commonly referred to as the degrees of freedom of the structure. It is necessary to consider all the independent degrees of freedom while writing the equilibrium equations.

These degrees of freedom are specified at supports, joints and at the free ends. For example, a propped cantilever beam (see Fig.14.01a) under the action of load P will undergo only rotation at B if axial deformation is neglected. In this case kinematic degree of freedom of the beam is only one i.e. ϑ_B as shown in the figure. In Figur 14.01 (b), we have nodes at *A,B,C* and *D*. Under the action of lateral loads, P_1, $P_2$and P_3 , this continuous beam deform as shown in the figure. Here axial deformations are neglected. For this beam we have five degrees of freedom ϑ_A, ϑ_B, ϑ_B, ϑ_D and $_D$ as indicated in the figure. In Fig.14.02a, a symmetrical plane frame is loaded symmetrically. In this case we have only two degrees of freedom ϑ_B and ϑ_C. Now consider a frame as shown in Fig.14.02b. It has three degrees of freedom viz. ϑ_B, ϑ_C and $_D$ as shown. Under the action of horizontal and vertical load, the frame will be displaced as shown in the figure. It is observed that nodes at *B* and *C* undergo rotation and also get displaced horizontally by an equal amount.

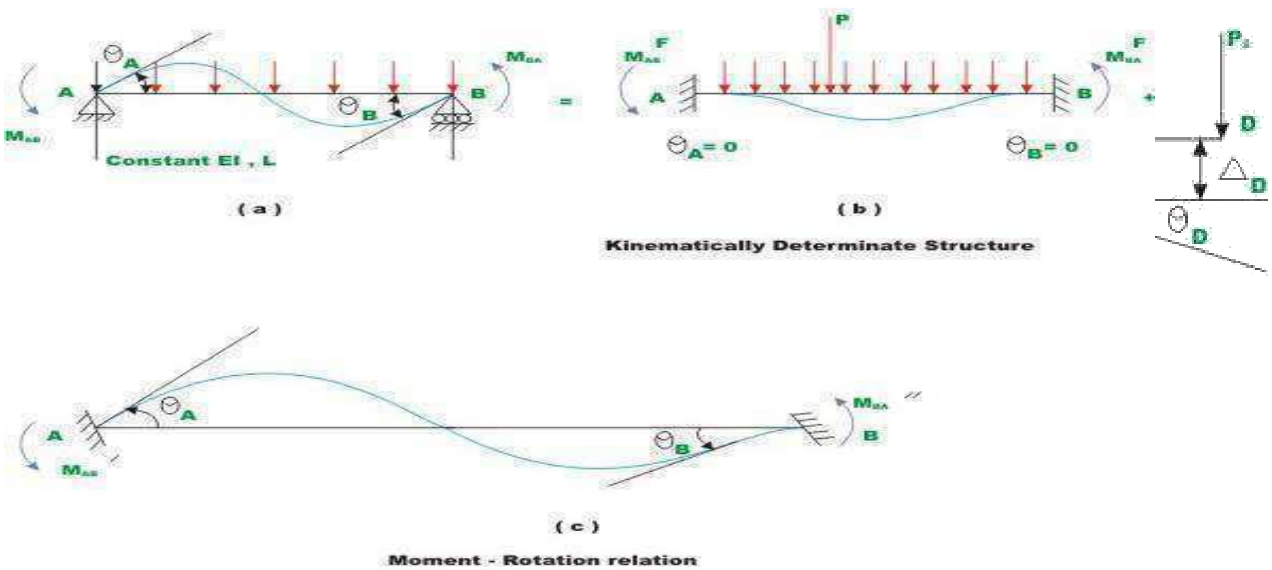


Figure 14.01:
Derivation of slope– deflection equation.
Hence in plane structures, each node can have at the most one linear displacement and one rotation. In this module first slope-deflection equations as applied to beams and rigid frames will be discussed.

Slope-Deflection Equations
Consider a typical span of a continuous beam *AB* as shown in Fig.14.1.The beam has constant flexural rigidity *EI* and is subjected to uniformly distributed loading and concentrated loads as shown in the figure. The beam is kinematic ally indeterminate to second degree. In this lesson, the slope-deflection equations are derived for the simplest case i.e. for the case of continuous beams with unyielding supports. In the next lesson, the support settlements are included in the slope-deflection equations.

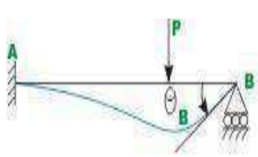


Fig. 14.01

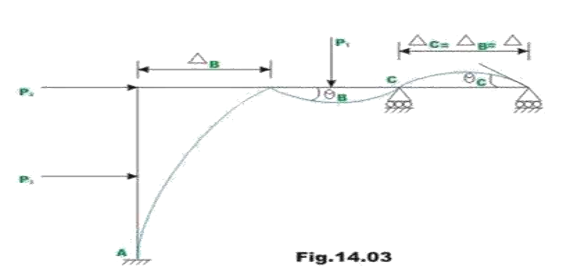


Fig.14.03

For this problem, it is required to derive relation between the joint end moments M_{AB} and M_{BA} in terms of joint rotations ϑ_A and ϑ_B and loads acting on the beam .Two subscripts are used to denote end moments. For example, end moments *MAB* denote moment acting at joint *A* of the member *AB*. Rotations of the tangent to the elastic curve are denoted by one subscript. Thus, ϑ_A denotes the rotation of the tangent to the elastic curve at A. The following sign conventions are used in the slope-deflection equations (1) Moments acting at the ends of the member in counterclockwise direction are taken to be positive. (2) The rotation of the tangent to the elastic curve is taken to be positive when the tangent to the elastic curve has rotated in the counterclockwise direction from its original direction. The slope-deflection equations are derived by superimposing the end moments developed due to (1) applied loads (2) rotation ϑ_A (3)

Rotation ϑ_B. This is shown in Fig.14.2 (a)-(c). In Fig. 14.2(b) a kinematic ally determinate structure is obtained. This condition is obtained by modifying the support conditions to fixed so that the unknown joint rotations become zero. The structure shown in Fig.14.2 (b) is known as kinematic ally determinate structure or restrained structure. For this case, the end moments are denoted by M_{AB}^F and M_{BA}^F. The fixed end moments are evaluated by force–method of analysis as discussed in the previous module. For example for fixed- fixed beam subjected to uniformly distributed load, the fixed-end moments are shown in Fig.14.3.

The fixed end moments are required for various load cases. For ease of calculations, fixed end forces for various load cases are given at the end of this lesson. In the actual structure end *A* rotates by ϑ_A and end *B* rotates by ϑ_B. Now it is required to derive a relation relating ϑ_A and ϑ_B with the end moments M'_{AB} and M'_{BA}. Towards this end, now consider a simply supported beam acted by moment M_{AB}' at *A* as shown in Fig. 14.4. The end moment M_{AB}' deflects the beam as shown in the figure. The rotations ϑ_A' and ϑ_B' are calculated from moment-area theorem.

$\Theta'_A = (M'_{AB}*L)(3EI)$ 3.1a
$\Theta'_B = (M'_{AB}*L)(6EI)$ 3.1b

Now a similar relation may be derived if only M_{BA}' is acting at end *B* (see Fig. 14.4).
$\Theta''_B = (M'_{BA}*L)(3EI)$ 3.2a
$\Theta''_A = (M'_{BA}*L)(6EI)$ 3.2b

Now combining these two relations, we could relate end moments acting at *A* and *B* to rotations produced at *A* and *B* as (see Fig. 14.2c)
$\Theta_A = (M'_{AB}*L)(3EI) - (M'_{AB}*L)/(6EI)$ 3.3a
$\Theta_B = (M'_{BA}*L)(3EI) - (M'_{BA}*L)/(6EI)$ 3.3b

Solving for M_{AB} and M_{BA} in terms of θ_A and θ_B,
$M'_{AB} = (2EI/L)(2\theta_A + \theta_B)$ 3.4
$M'_{BA} = (2EI/L)(2\theta_B + \theta_A)$ 3.5
Now writing the equilibrium equation for joint moment at *A* (see Fig. 14.2).
$M_{AB} = M^F_{AB} + M'_{AB}$3.6a
Similarly writing equilibrium equation for joint B

$M_{BA} = M^F_{BA} + M'_{BA}$3.6b

Substituting the values of M'_{AB} and M'_{BA}

$M_{AB} = M^F_{AB} + (2EI/L)(2\theta_A + \theta_B)$ 3.7a

$M_{BA} = M^F_{BA} + (2EI/L)(2\theta_B + \theta_A)$ 3.7b

Sometimes one end is referred to as near end and the other end as the far end .In that case, the above equation may be stated as the internal moment at the near end of the span is equal to the fixed end moment at the near end due to external loads plus $\frac{2EI}{L}$ times the sum of twice the slope at the near end and the slope at the far end. The above two equations (14.7a) and (14.7b) simply referred to as slope–deflection equations. The slope-deflection equation is nothing but a load displacement relationship.

3.3 Application of Slope-Deflection Equations to Statically Indeterminate Beams

The procedure is the same whether it is applied to beams or frames. It may be summarized as follows:

1) Identify all kinematic degrees of freedom for the given problem. This can be done by drawing the deflection shape of the structure. All degrees of freedom are treated as unknowns in slope-deflection method.
2) Determine the fixed end moments at each end of the span to applied load. The table given at the end of this lesson may be used for this purpose.
3) Express all internal end moments in terms of fixed end moments and near end, and far end joint rotations by slope-deflection equations.
4) Write down one equilibrium equation for each unknown joint rotation. For example, at a support in a continuous beam, the sum of all moments corresponding to an unknown joint rotation at that support must be zero.
5) Write down as many equilibrium equations as there are unknown joint rotations.
6) Solve the above set of equilibrium equations for joint rotations.
7) Now substituting these joint rotations in the slope-deflection equations evaluate the end moments.
8) Determine all rotations.

Example

A continuous beam *ABC* is carrying uniformly distributed load of 2 kN/m in addition to a concentrated load of 20 kN as shown in Fig.14.5a. Draw bending moment and shear force diagrams. Assume *EI* to be constant.

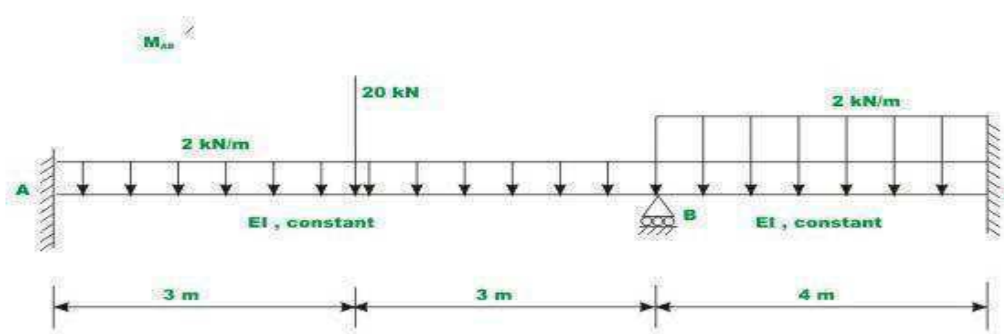


Fig. 14.5(a) Example 14.1

(a) Degrees of freedom

It is observed that the continuous beam is kinematic ally indeterminate to first degree as only one joint rotation ϑ_b is unknown. The deflected shape / elastic curve of the beam is drawn in Fig.14.5b in order to identify degrees of freedom.
By fixing the support or restraining the support *B* against rotation, the fixed-fixed beams area obtained as shown in Fig. C

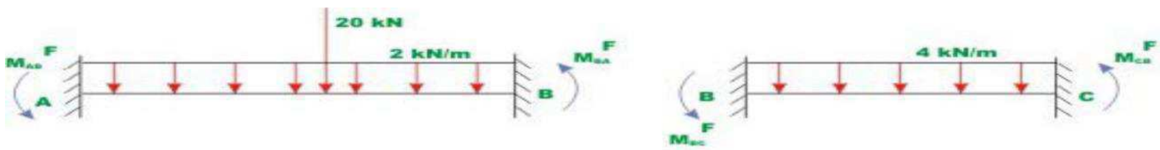


Fig. 14.5 (c) Restrained Structure.

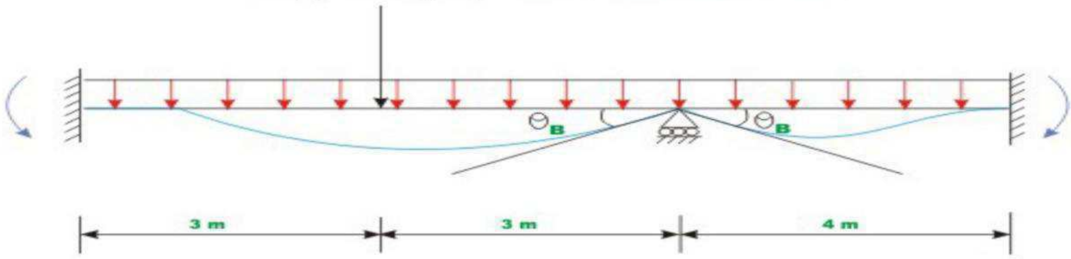


Fig. 14.5 (b) Elastic curve of the beam with unknown displacement component Θ_B

(b). Fixed end moments M_{AB}^F,M_{BA}^F, M_{BC}^F and M_{CB}^F are calculated referring to the Fig. 14 and following the sign conventions that counterclockwise moments are positive.

M^F_{AB} = (2*6*6)/12 + (20*3*3*3)(6*6) = 21 kN M.
M^F_{BA} = - 21 kN M.
M^F_{BC} = (4*4)/12 = 5.33 kN M.
M^F_{BA} = - 5.33 kN M.

(c) Slope-deflection equations
Since ends *A* and *C* are fixed, the rotation at the fixed supports is zero, $\vartheta_A = \vartheta_C = 0$. Only one non-zero rotation is to be evaluated for this problem. Now ,write slope-deflection equations for span *AB* and *BC*.
$M_{AB} = M^F_{AB} + (2EI/L)(2\Theta_A + \theta_B) = 21 + (2EI/6)\,\theta_B$
$M_{BA} = M^F_{BA} + (2EI/L)(2\Theta_B + \theta_A) = -21 + (4EI/6)\,\theta_B$
$M_{BC} = 5.33 + EI\theta_B$
$M_{CB} = -5.33 + 0.5EI\theta_B$

(d) Equilibrium equations
In the above four equations (2-5), the member end moments are expressed in terms of unknown rotation ϑ_B. Now, the required equation to solve for the rotation ϑ_B is the moment equilibrium equation at support *B*. The free body diagram of support *B* along with the support moments acting on it is shown in Fig. 14.5d. For, moment equilibrium at support *B*, one must have,

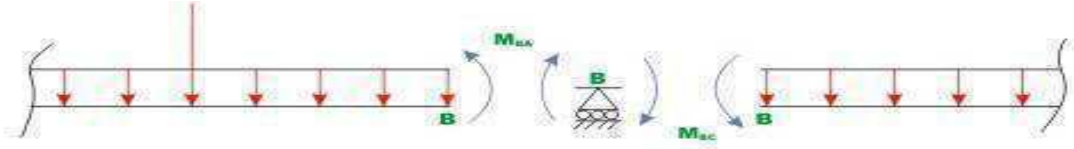


Fig. 14.5 d Free- body diagram of the joint B

$\Sigma M_B = 0 \quad M_{BA} + M_{BC} = 0$

Substituting the values of M_{BA} and M_{BC} in the above equilibrium equation,

$-21 + (4EI/6)\,\theta_B + 5.33 + EI\theta_B = 0$

$\theta_B = 9.4/(EI)$

(e) End moments

After evaluating θ_B, substitute it in equations (2-5) to evaluate beam end moments. Thus,

$M_{AB} = 24.133$ **kN M.**

$M_{BA} = -\,14.733$ **kN M.**

$M_{BC} = 14.733$ **kN M.**

$M_{CB} = -\,0.63$ **kN M.**

(f) Reactions

Now, reactions at supports are evaluated using equilibrium equations (vide Fig. 14.5e)

$R_A \times 6 + 14.733 - 20 \times 3 - 2 \times 6 \times 3 - 24.133 = 0$

$R_A = 17.567$ kN(↑)

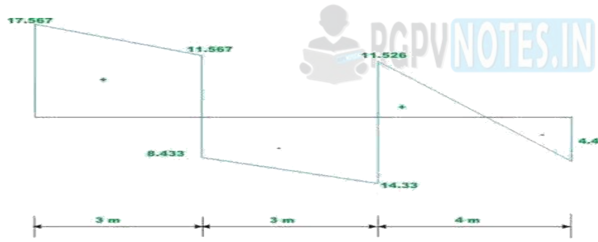
$R_{BL} = 20 + 12 - 17.567 = 14.433$ kN(↑)

$R_{BR} * 4 - 14.733 - (4*4*2) + 0.63 = 0$

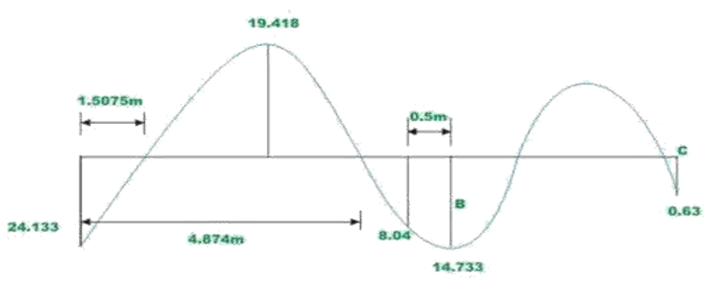
$R_{BR} = 11.526$ kN(↑)

$R_C = 16 - 11.526 = 4.47$ kN(↑)

The shear force and bending moment diagrams are shown in Fig. 14.5f.



Shear force diagram



Bending Moment diagram

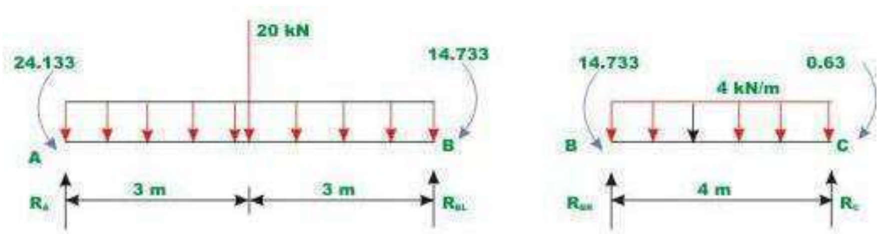


Fig. 14.5 (e) Free - body diagram of two members

Example

Draw shear force and bending moment diagram for the continuous beam *ABCD* loaded as shown in Fig.14.6a.The relative stiffness of each span of the beam is also shown in the figure.

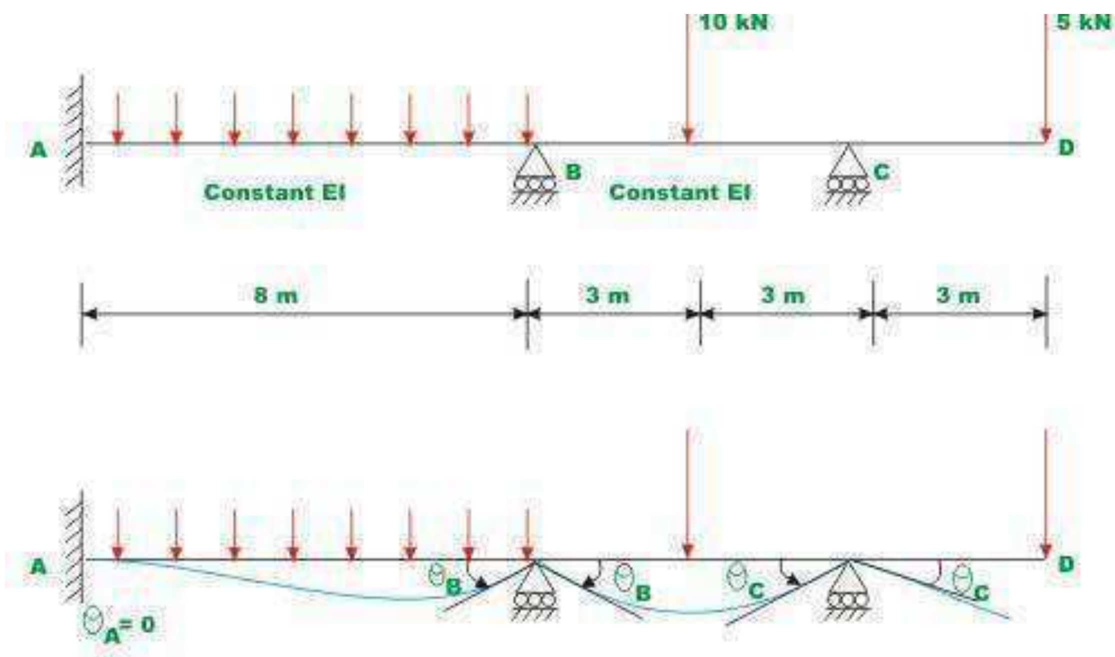


Fig. 14.6a Continuous beam of Example 14.2

For the cantilever beam portion *CD*, no slope-deflection equation need to be written as there is no internal moment at end *D*. First, fixing the supports at *B* and *C*, calculate the fixed end moments for span *AB* and *BC*. Thus,

M^F_{AB} = (3*8*8)/12 = 16 kN M.
M^F_{BA} = - 16 kN M.
M^F_{BC} = (10*3*3)(6*6) = 7.5 kN M.
M^F_{BA} = - 7.5 kN M.

In the next step write slope-deflection equation. There are two equations for each span of the

continuous beam.

M_{AB} = 16 + (2EI/L) θ_B = 16 + 0.25EI θ_B

M_{AA} = - 16 + 0.5EI θ_B

M_{BC} = 7.5 + (2EI/L) (2θ_B + θ_C) = 7.5 + 1.334EI θ_B + 0.667EI θ_C

M_{CB} = - 7.5 + 1.334EI θ_C + 0.667EI θ_B

Equilibrium equations

The free body diagram of members *AB*,*BC* and joints *B* and *C* are shown in Fig.14.6b.One could write one equilibrium equation for each joint B and C.

Fig. 14.6 b Free - body diagrams of joints B and C along with members

Support B,

$\Sigma M_B = 0$ $M_{BA} + M_{BC} = 0$

$\Sigma M_C = 0$ $M_{BC} + M_{CD} = 0$

We know that M_{CD} = 15 kN.M

M_{CB} = 15 kN.M. Substituting the values of M_{CB} and M_{CD} in the above equation we get

θ_B = 8.164 and θ_C = 9.704

Substituting θ_B $\theta_C$in the slope-deflection equations, we get

MAB = 18.04 kN M.

MBA = 11.918 kN M.

MBC = - 11.918 kN M.

MCB = - 15.0 kN M.

Reactions are obtained from equilibrium equations (ref. Fig. 14.6c)

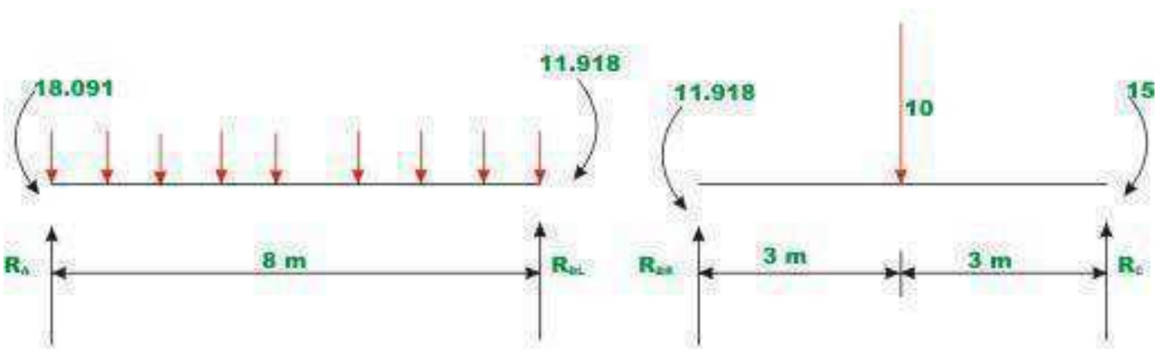


Fig. 14.6 c Computation of reactions

$R_A \times 8 - 18.041 - 3 \times 8 \times 4 + 11.918 = 0$

R_A=12.765 kN
R_{BR}=5−0.514kN =4.486 kN
R_{BL}=11.235 kN
R_C=5+0.514kN =5.514 kN
The shear force and bending moment diagrams are shown in Fig. 14.6d.

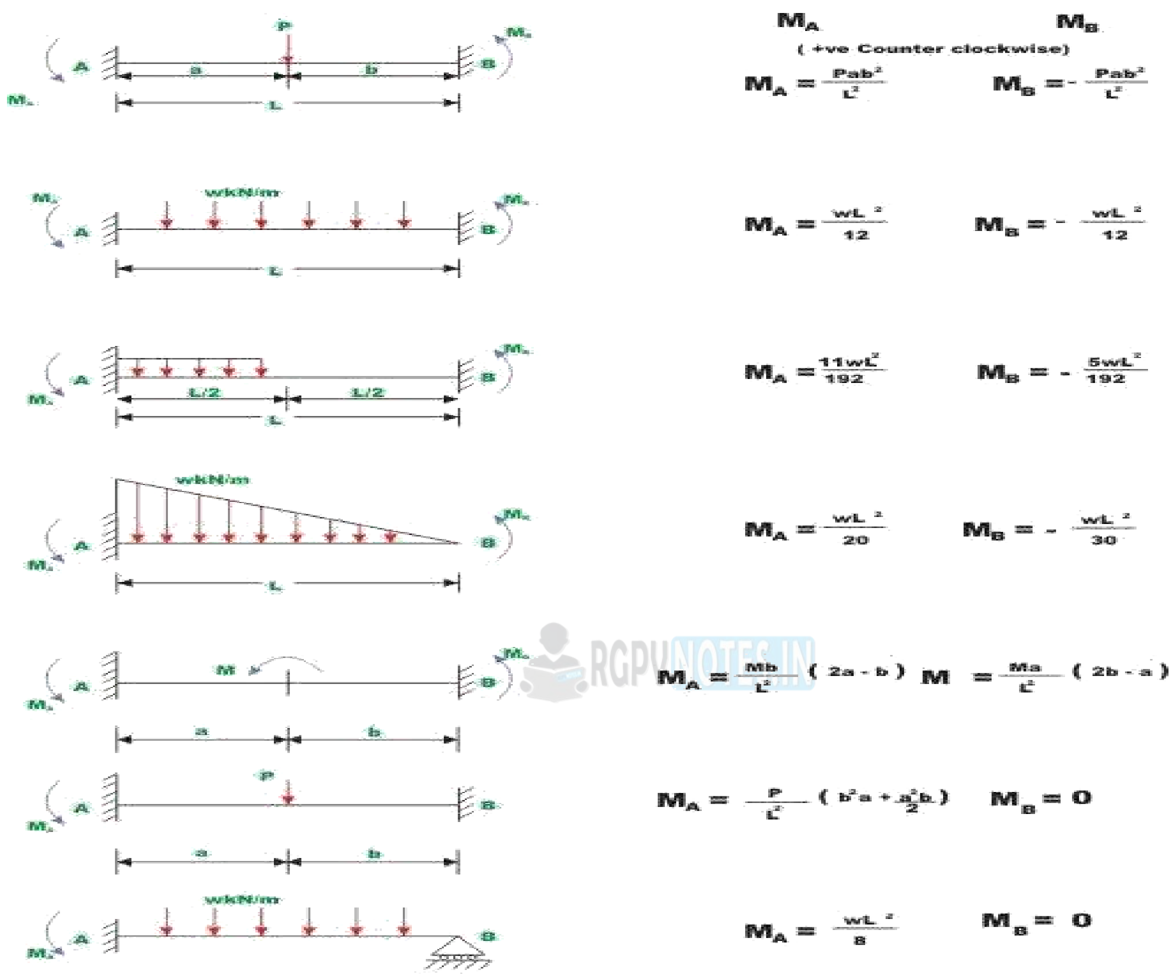
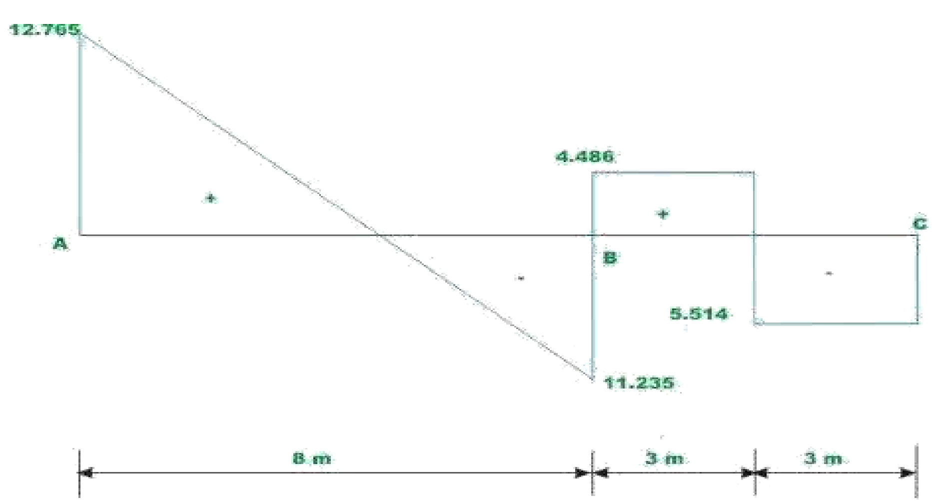


Fig. 14.7 Table of fixed end moments



For ease of calculations, fixed end forces

Summary

In this lesson the slope-deflection equations are derived for beams with unyielding supports. The kinematic ally indeterminate beams are analyzed by slope-deflection equations. The advantages of displacement method of analysis over force method of analysis are clearly brought out here. A couple of examples are solved to illustrate the slope-deflection equations.

Column Analogy Method

Introduction

The method is based on a mathematical similarity (i.e. analogy) between the stresses developed on a column section subjected to eccentric load and the moments imposed on a member due to fixity of its supports.

In the analysis of actual engineering structures of modern times, so many analogies are used like slab analogy, and shell analogy etc. In all these methods, calculations are not made directly on the actual structure but, in fact it is always assumed that the actual structure has been replaced by its mathematical model and the calculations are made on the model. The final results are related to the actual structure through same logical engineering interpretation.

In the method of column analogy, the actual structure is considered under the action of applied loads and the redundant acting simultaneously on a BDS. The load on the top of the analogous column is usually the B.M.D. due to applied loads on simple spans and therefore the reaction to this applied load is the B.M.D. due to redundant on simple spans considers the following fixed ended loaded beam.

The resultant of B.M.D's due to applied loads does not fall on the midpoint of analogous column section which is eccentrically loaded.
Ms diagram = BDS moment diagram due to applied loads.
Mi diagram = Indeterminate moment diagram due to redundant.

If we plot (+ve) B.M.D. above the zero line and (−ve) B.M.D below the zero line (both on compression sides due to two sets of loads) then we can say that these diagrams have been plotted on the compression side. (The conditions from which MA & MB can be determined, when the method of consistent deformation is used, are as follows). From the Geometry requirements, we know that

(1) The change of slope between points A & B = 0; or sum of area of moment diagrams between A & B = 0 (note that EI = Constant), or area of moment diagrams of figure b = area of moment diagram of figure c.
(2) The deviation of point B from tangent at A = 0; or sum of moment of moment diagrams between A & B about B = 0, or Moment of moment diagram of figure(b) about B = moment of moment diagram of figure (c) about B. Above two requirements can be stated as follows.
(1) Total load on the top is equal to the total pressure at the bottom and;

(2) Moment of load about B is equal to the moment of pressure about B), indicates that the analogous column is on equilibrium under the action of applied loads and the redundant.

7.1. SIGN CONVENTIONS:– It is necessary to establish a sign convention regarding the nature of the applied load (Ms –diagram) and the pressures acting at the base of the analogous column (Mi–diagram.)

1. Load (P) on top of the analogous column is downward if Ms/EI diagram is (+ve) which means that it causes compression on the outside or (sagging) in BDS vice-versa. If EI is constant, it can be taken equal to units.

2. Upward pressure on bottom of the analogous column (Mi –diagram) is considered as (+ve).

3. Moment (M) at any point of the given indeterminate structure (maximum to 3^rd^degree) is given by the formula.

M = Ms –Mi, which is (+ve) if it causes compression on the outside of members

In the last lesson, slope-deflection equations were derived without considering the rotation of the beam axis. In this lesson, slope-deflection equations are derived considering the rotation of beam axis. In statically indeterminate structures, the beam axis rotates due to support yielding and this would in turn induce reactions and stresses in the structure. Hence, in this case the beam end moments are related to rotations, applied loads and beam axes rotation. After deriving the slope-deflection equation in section 15.2, few problems are solved to illustrate the procedure.

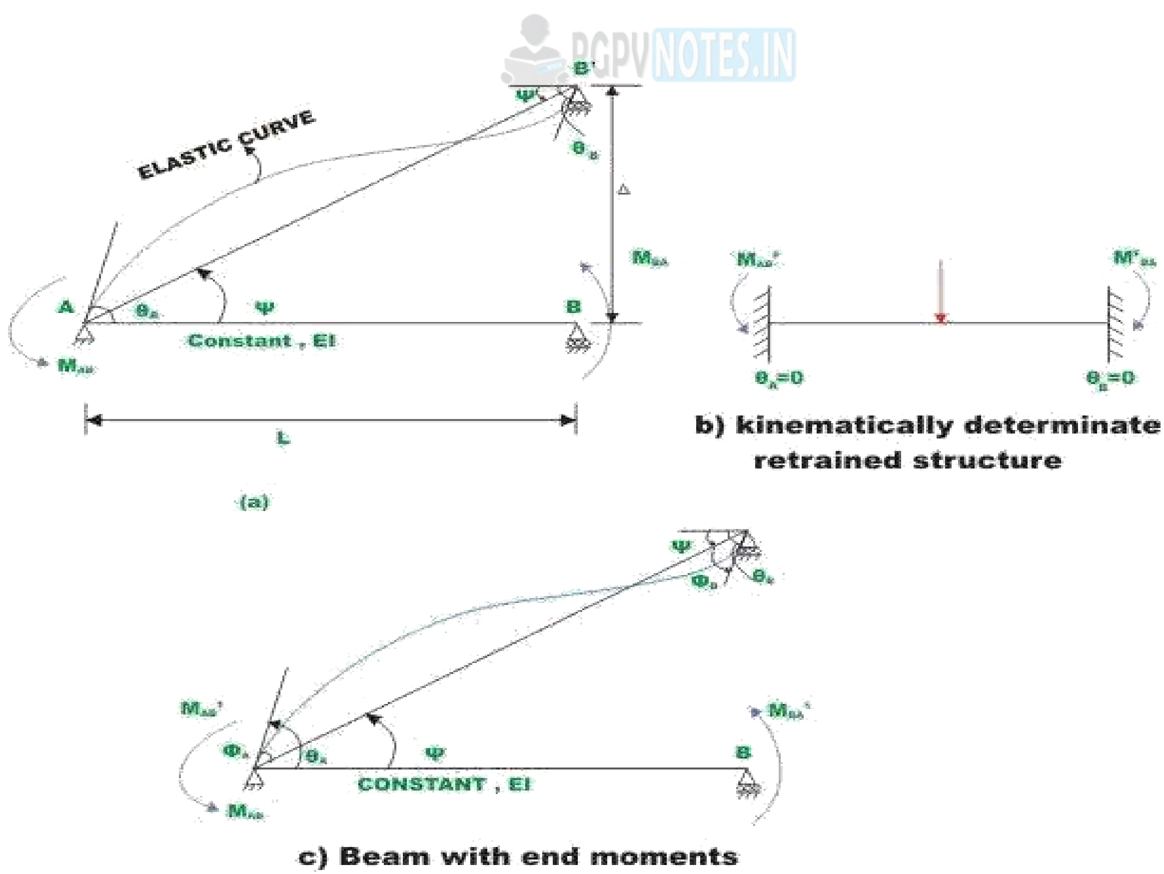


Figure 15.1

Now superposing the fixed end moments due to external load and end moments due to displacements, the end moments in the actual structure is obtained .Thus (see Fig.15.1)

In the above equations, it is important to adopt consistent sign convention. In the above derivation is taken to be negative for downward displacements. In the continuous beam *ABC*, two rotations ϑ_B and ϑ_C need to be evaluated.

Hence, beam is kinematic ally indeterminate to second degree. As there is no external load on the beam, the fixed end moments in the restrained beam are zero (see Fig.15.2b).

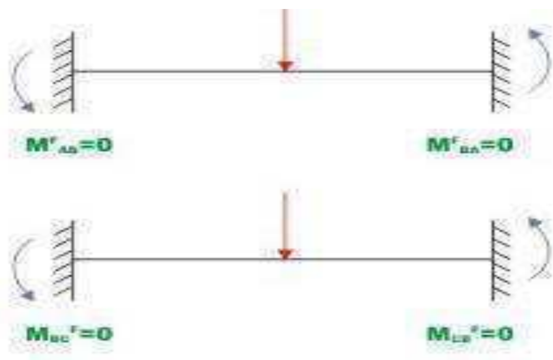


Figure 15.2 (b)

For each span, two slope-deflection equations need to be written. In span *AB*,*B* is below *A*. Hence, the chord *AB* rotates in clockwise direction. Thus, ψ_{AB} *is* taken as negative .In span *BC*, the support *C* is above support *B*, Hence the chord joining *B′C* rotates in anticlockwise direction.

$\psi_{BC} = \psi_{CB} = 1\times10^{-3}$

Writing slope-deflection equations for span *BC*,

$M_{BC} = 0.8EI\vartheta_B + 0.4EI\vartheta_C - 1.2\times10^{-3}\,EI$

$M_{CB} = 0.8EI\vartheta_C + 0.4EI\vartheta_B - 1.2\times10^{-3}\,EI$

Now, consider the joint equilibrium of support *B* (see Fig.15.2c)

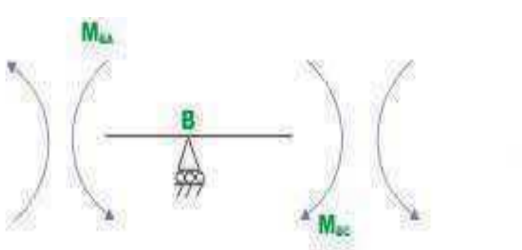
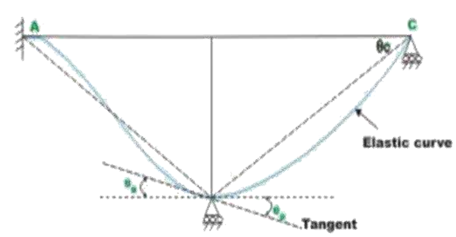


Fig 15.2c Free body diagram of joint B



15.2 f Elasctic curve

$M_{BA} + M_{BC} = 0$

Substituting the values of M_{BA} and M_{BC} in equation (6),
$0.8EI\vartheta_B + 1.2\times10^{-3}EI + 0.8EI\vartheta_B + 0.4EI\vartheta_C - 1.2\times10^{-3}EI = 0$
Simplifying, $1.6\vartheta_B + 0.4\vartheta_C = 1.2\times10^{-3}$
Also, the support *C* is simply supported and hence, $M_{CB} = 0$
$M_{CB} = 0 = 0.8\vartheta_C + 0.4\vartheta_B - 1.2\times10^{-3}\,EI$
$0.8\vartheta_C + 0.4\vartheta_B = 1.2\times10^{-3}$

We have two unknowns ϑ_B and ϑ_C and there are two equations in ϑ_B and ϑ_C. Solving equations (7) and (8)

$$\vartheta_B = -0.4286\times10^{-3} \text{ radians}$$

$$\vartheta_C = 1.7143\times10^{-3} \text{ radians} \qquad (9)$$

Substituting the values of ϑ_B, ϑ_C and EI in slope-deflection equations,
$M_{AB} = 82.285$ kN.m $M_{BA} = 68.570$ kN.m
$M_{BC} = -68.573$ kN.m $M_{CB} = 0$ kN.m

Reactions are obtained from equations of static equilibrium (vide Fig.15.2d)
In beam *AB*,
$\sum M_B = 0, R_A = 30.171$ kN(↑)
$R_{BL} = -30.171$ kN(↓)
$R_{BR} = -13.714$ kN(↓)
$R_C = 13.714$ kN(↑)
The sheer force and bending moment diagram is shown in Fig.15.2e and elastic curve is shown in Fig.15.2f.

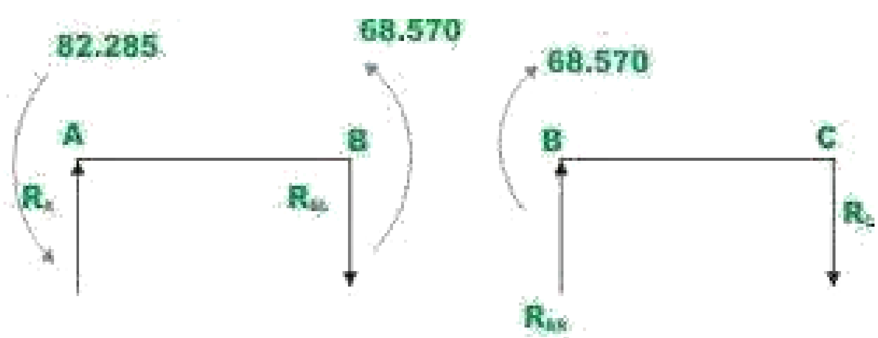


Fig 15.2d Computation of reactions

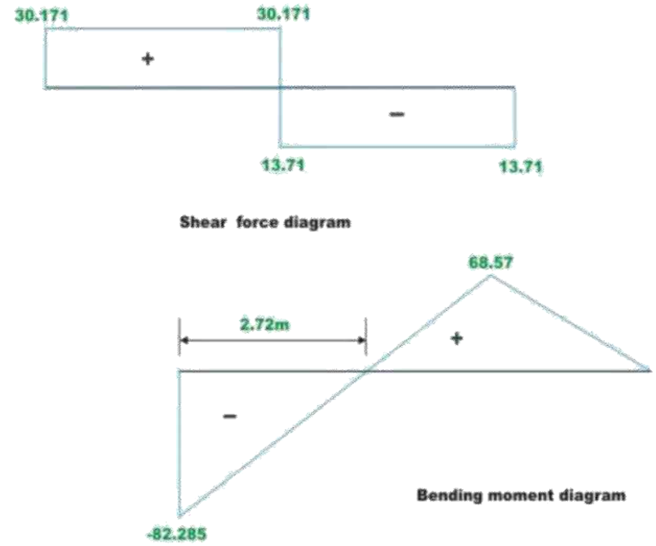


Figure 15.2e Shear force and bending moment diagram

Example

A continuous beam *ABCD* is carrying a uniformly distributed load of 5 kN/m as shown in Fig. 15.3a. Compute reactions and draw shear force and bending moment diagram due to following support settlements.

Support *B* 0.005m vertically downwards
Support *C* 0.01 m vertically downwards
Assume E =200 GPa, I = 1.35 ×10^{-3} m^4

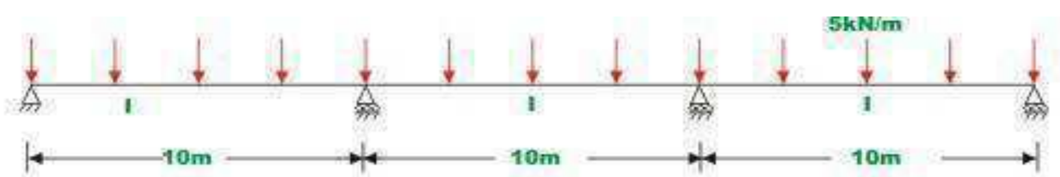


Fig 15.3a Continuous beam of Example 15.2

In the above continuous beam, four rotations ϑ_A, θ_B, θ_c and θ_d are to be evaluated. One equilibrium equation can be written at each support. Hence, solving the four equilibrium equations, the rotations are evaluated and hence the moments from slope-deflection equations. Now consider the kinematic ally restrained beam as shown in Fig.15.3b.

Referring to standard tables the fixed end moments may be evaluated .Otherwise one could obtain fixed end moments from force method of analysis. The fixed end moments in the present case are (vide fig.15.3b)

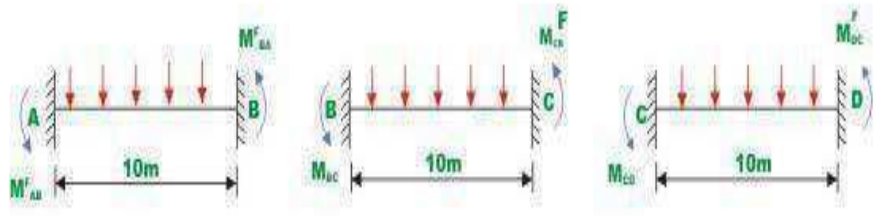


Fig 15.3b Kinematically restrained beam

M_{AB}^F=41.667 kN.m
M_{BA}^F= −41.667 kN.m(clockwise)
M_{BC}^F =41.667 kN.m(counterclockwise)
M_{CB}^F= −41.667 kN.m(clockwise)
M_{CD}^F= 41.667 kN.m(counter clockwise)
M_{DC}^F= −41.667 kN.m(clockwise)

In the next step, write slope-deflection equations for each span. For the span *AB*,*B* is below *A* and hence the chord joining *AB*′ rotates in the clockwise direction (see Fig.15.3c)

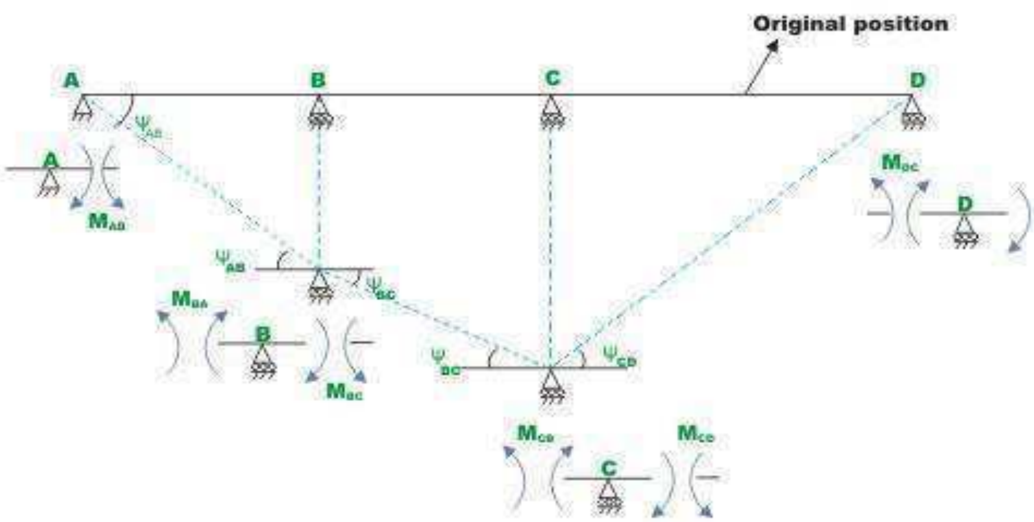


Fig 15.3c New support positions and free body diagrams of support

Now, writing the expressions for the span end moments, for each of the spans,

$M_{AB}=41.667+0.2EI(2\vartheta_A+\vartheta_B+0.0005)$

$M_{BA}= -41.667+0.2EI(2\vartheta_B+\vartheta_A+0.0005)$

$M_{BC}=41.667+0.2EI(2\vartheta_B+\vartheta_C+0.0005)$

$M_{CB}= -41.667+0.2EI(2\vartheta_C+\vartheta_B+0.0005)$

$M_{CD}=41.667+0.2EI(2\vartheta_C+\vartheta_D-0.001)$

$M_{DC}= -41.667+0.2EI(2\vartheta_D+\vartheta_C-0.001)$ (3)

For the present problem, four joint equilibrium equations can be written, one each for each of the supports. They are

1. $M_A=0\Rightarrow M_{AB}=0$
2. $M_B=0\Rightarrow M_{BA}+M_{BC}=0$
3. $M_C=0\Rightarrow M_{CB}+M_{CD}=0$

$\sum M_D = 0\Rightarrow M_{DC}= 0$ (4)

Substituting the values of beam end moments from equations (3) in equation (4), four equations are obtained in four unknown rotations ϑ_A ,ϑ_B ,ϑ_C and ϑ_D .

Theyare,

(EI = $200\times10^3\times1.35\times10^{-6}$ =270,000 kN.m^2)

$2\vartheta_A+\vartheta_B= -1.2716\times10^{-3}$

$\vartheta_A+4\vartheta_B+\vartheta_C= -0.001$

$\vartheta_B+4\vartheta_C+\vartheta_D=0.0005$

$\vartheta_C+2\vartheta_D=1.7716\times10^{-3}$

Solving the above sets of simultaneous equations, values of ϑ_A, θ_B, θ_C and ϑ_D are evaluated. Substituting the values in slope-deflection equations the beam end moments are evaluated.

$M_{AB}=41.667+0.2\times270{,}000\{2(-5.9629\times10^{-4})+(-7.9013\times10^{-5})+0.0005)\}=0$

$M_{BA}= -41.667+0.2\times270{,}000\{2(-7.9013\times10^{-5})-5.9629\times10^{-4}+0.0005\}= -55.40$ kN.m

$M_{BC}=41.667+0.2\times270{,}000\{2(-7.9013\times10^{-5})+(-8.7653\times10^{-5})+0.0005\}=55.40$ kN.m

$M_{CB}= -41.667+0.2\times270{,}000\{2(-8.765\times10^{-5})-7.9013\times10^{-5}+0.0005\}= -28.40$ kN.m

$M_{CD}=41.667+0.2\times270{,}000\{2\times(-8.765\times10^{-5})+9.2963\times10^{-4}-0.001\}=28.40$ kN.m

$M_{DC} = -41.667+0.2\times270{,}000\{2\times9.2963\times10^{-4}-8.7653\times10^{-5}-0.001\}=0$ kN.m

Reactions are obtained from equilibrium equations. Now consider the free body diagram of the beam with end moments and external loads as shown in Fig.15.3d.

R_A=19.46 kN(↑)

R_{BL}=30.54 kN(↑)

R_{BR}=27.7 kN(↑)

R_{CL}=22.3 kN(↑)

R_{CR}=27.84 kN(↑)

R_D=22.16 kN(↑)

The sheer force and bending moment diagrams are shown in Fig.15.5e.

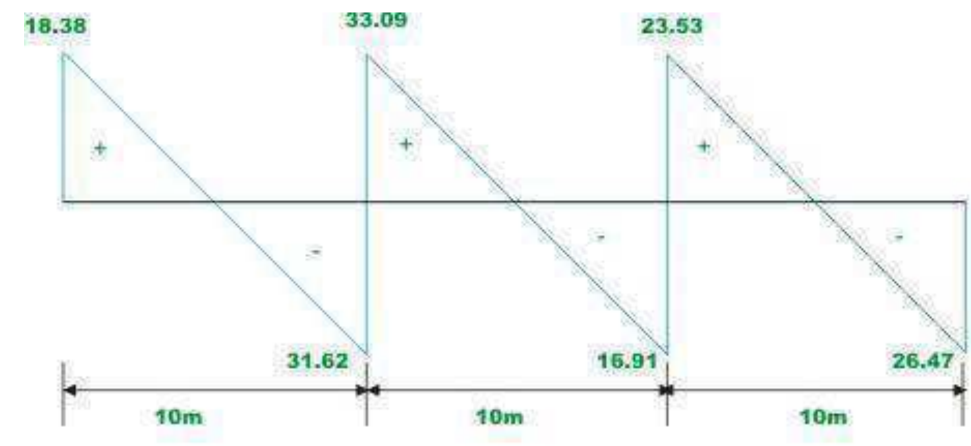


Fig 15.3d Shear force diagram

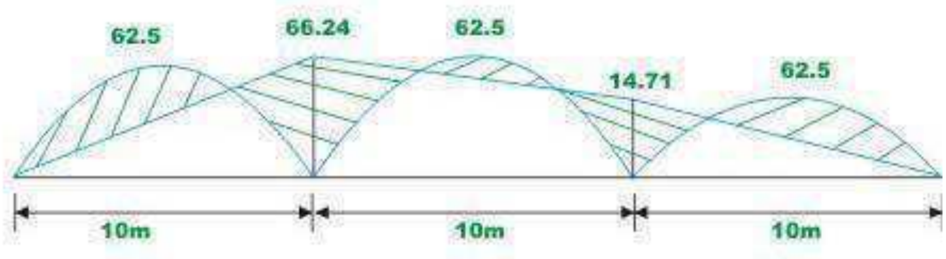


Fig. 15.3e Bending moment diagram

Summary

In this lesson, slope-deflection equations are derived for the case of beam with yielding supports. Moments developed at the ends are related to rotations and support settlements. The equilibrium equations are written at each support. The continuous beam is solved using slope-deflection equations. The deflected shape of the beam is sketched. The bending moment and shear force diagrams are drawn for the examples solved in this lesson.

